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Other stylized facts:-

Facts 5 - 7 - Read text.

~~Reg~~ :

Harrod Domar Model

{ 1939
Roy Harrod
Evsey Dommar }

There were 3 sets of issues that was raised by Harrod:

First, the possibility of steady growth in a model of fixed capital - output ratio C and a fixed savings - output ratio s is noted. A unit of capital will produce $\frac{1}{C}$ unit of output, which in turn will generate $\frac{s}{C}$ units of net savings. So that the rate of growth of capital stock will equal s/C. Since output is proportional to capital, that will be the rate of growth of output as well.

Harrods second point is concerned with showing the instability of this path of steady growth. In the simple Harrod - Domar

model It is assumed that invst is determined entirely by planned savings and there is no independent invst function based on the expectation of the future which of course, rather pre - keynesian in its approach and may not make much sense in an

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an unplanned economy'

Harrod's qn are the following:-

(a) When will the investor's expectations be realized ?, and
(b) What will happen if they are not realised ?

As mentioned earlier the model consists of a fixed capital-output ratio C and fixed savings output ratio s.

ie $\boxed{C = \frac{K}{Y}}$ and $\boxed{s = \frac{S}{Y}}$

As it was also mentioned that in this model it is assumed that invst is determined entirely by "planned" savings

~~X~~ hence $\boxed{s = \frac{S}{Y}}$ is the invst rate as well → pre keynesian approach.

$\frac{K}{Y} = C \Rightarrow Y = \frac{K}{C}$

$\Rightarrow \Delta Y = \frac{\Delta K}{C}$ (as C is a const)

$\Rightarrow \frac{\Delta Y}{Y} = \frac{\Delta K / C}{Y}$

$\Rightarrow \frac{\Delta Y}{Y} = \frac{\Delta K / C}{K / C}$ (as $Y = \frac{K}{C}$)

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$$\Rightarrow \boxed{\frac{\Delta Y}{Y} = \frac{\Delta K}{K}}$$

ie Growth rate of output equals that of capital this is only true as mentioned earlier due to the fact that output is proportional to capital

$\boxed{\text{ie } Y = \frac{K}{C}}$.

we know

$Y = \frac{K}{C}$ (i) and $\frac{S}{Y} = s \Rightarrow S = sY$.

$S = s \cdot \frac{K}{C}$ (from (i)). —— (ii)

Also we know $\frac{\Delta Y}{Y} = \frac{\Delta K}{K}$ —— (iii)

from (ii) $\boxed{S = \frac{s}{C} \cdot K = \Delta K}$. (As change in cpl stock (I) equals savings in Harrod model.)

Thus (iii) becomes $\boxed{\frac{\Delta Y}{Y} = \frac{s}{C} = \frac{\Delta K}{K}}$.

Suppose $s = 0.25$
$C = 5$.

Then $\frac{\Delta Y}{Y} = \frac{0.25}{5} \Rightarrow \Delta Y = 0.05\,Y$

ie Growth rate of Y(output) is @ 5%.

As said this model is realistic for planned economies. Harrod thus introduced an investment function of the 'accelerator' type.

Expectations of addl effective demand determines the level of investment in the Harrod model and that investment, through the multiplier generates a certain level of effective demand in a Keynesian manner.

Taking expected demand in period t - X_t.

Period	Actual demand	Cpl Stock
t-1	Y_{t-1}	cY_{t-1}
t	X_t	cX_t

⇒ Expected addl flow of output

$$= X_t - Y_{t-1}$$

~~and investment in this~~

Invst in this accelerator model in time 't' ie I_t will be C times expected addl flow of output

Hence $I_t = C(X_t - Y_{t-1})$.

(8) During these times we take X_t as $d\gamma$ but now it is Y_{t-1}.

Actual agg dd.

$$Y_t = \frac{I_t}{S} - \frac{C}{S}(X_t - Y_{t-1})$$

(We know that $\Delta Y = \frac{A}{1-c}\Delta I$ here $A = \bar{I}$ and $1-c = s$ hence $\frac{A}{1-c} = \frac{I}{s}$.

$$Y_t = \frac{X_t - Y_{t-1}}{S/C}$$

$$\frac{Y_t}{X_t} = \frac{(X_t - Y_{t-1})/X_t}{S/C} \quad \text{(eqn 3 in txtbook)}$$

where.

Expected rate of growth $= \hat{g}_t = \frac{X_t - Y_{t-1}}{X_t}$ (*)

$$\Rightarrow \boxed{\frac{Y_t}{X_t} = \frac{\hat{g}_t}{S/C}} \quad \text{(eqn 4 in txtbook)}$$

Thus it is obvious that expectations'll be realized ie X_t will equal Y_t iff the expected growth rate $\hat{g}_t$ equal S/C.

This rate of growth S/C which is realized if expected is what Harrod called the 'warranted' rate of growth — g_w.

[See — $\frac{Y_t}{X_t} = \hat{g}_t / S/C \Rightarrow Y_t = X_t \cdot \frac{\hat{g}_t}{S/C}$

$\Rightarrow Y_t = X_t \Leftrightarrow \hat{g}_t = S/C$]

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what if $\hat{g}_t \neq \frac{s}{c}$?

Defining actual rate of growth

$$g_t = \frac{Y_t - Y_{t-1}}{Y_t}$$

so

expected rate of growth $\hat{g}_t = \frac{X_t - Y_{t-1}}{X_t}$

$$\hat{g}_t = 1 - \frac{Y_{t-1}}{X_t}$$

$$\Rightarrow Y_{t-1} = X_t (1 - \hat{g}_t)$$

$$\Rightarrow \boxed{X_t = \frac{Y_{t-1}}{1 - \hat{g}_t}} //$$

11^{ly}

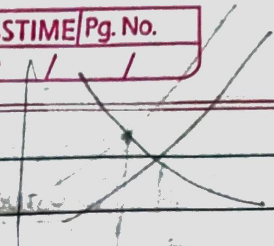
$$g_t = \frac{Y_t - Y_{t-1}}{Y_t}$$

$$= 1 - \frac{Y_{t-1}}{Y_t}$$

$$\Rightarrow \boxed{Y_t = \frac{Y_{t-1}}{1 - g_t}} //$$

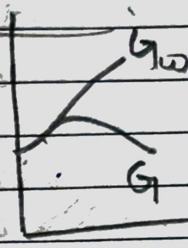
Accelerator $\frac{\Delta K}{\Delta Y}$

Lesser output ⇒ lesser capital ⇒ further move away from equilibrium



Thus $\frac{Y_t}{X_t} = \frac{\cancel{Y_{t-1}} / 1-g_t}{\cancel{Y_{t-1}} / 1-\hat{g}_t}$.

$\frac{Y_t}{X_t} = \frac{1-\hat{g}_t}{1-g_t}$. $\hat{g}_t / s/c$



If $\boxed{\hat{g}_t > s/c}$	If $\boxed{\hat{g}_t < s/c}$
then $1-\hat{g}_t > 1-g_t$	then $1-\hat{g}_t < 1-g_t$
(5):- ⇒ $\boxed{g_t > \hat{g}_t}$	⇒ $\boxed{g_t < \hat{g}_t}$
⇒ $\boxed{Y_t > X_t}$	⇒ $\boxed{Y_t < X_t}$.
(3):- Expectations less than actual	Expectations more than actual.
$SS < DD$. (⇒ Inflation)	$SS > DD$ Deflation

This is the beginning of Harrod's Instability problem.

(*) If the investors anticipate more than the warranted rate of growth s/c then the actual growth rate of demand will exceed even the high expected growth rate, so that instead of feeling that they expected too much they are likely to feel that they expected too little. (*) Same reasoning goes for the other way around. Hence market give perverse signal and thus we move further and further away from equilibrium.

A possible explanation for the same is the underlying AEH as this has been dealt in the previous semester (AEH's limitation for continuously T/ trends),

i.e $g_t - g'_{t-1} = \lambda(g_{t-1} - g'_{t-1})$ and hence the problem.

eg:

$s = 0.20$

$C = 2$

$\Rightarrow \frac{s}{c} = \frac{0.20}{2} = 10\% = 0.10$

Suppose $Y_{t-1} = 90$.

$\Rightarrow$ with $g_t = s/c = 10\%$.

$$Y_t = \frac{Y_{t-1}}{1 - g_t} = \frac{90}{1 - 0.10} = 100.$$

i.e if the current level of output is 90, then a 10% warranted growth rate will mean a movement to 100.

The $Y_t = 100$

$$I_t = 2(100 - 90) = 20$$

$$Y_t = \frac{20}{0.2} = 100!$$

Thus if inshrs in fact expect an output of 100 and they'll invest 20 units in trying to create a capacity for an add'l 10 units of dd. And this invst of 20 units 'll generate through the multiplier of 5 (implicit in a savings coefficient of 20%), a dd level of 100, so that expectations 'll be realized

Case A Take:

$x_t = 99$. $\frac{9}{99}$ $\qquad \hat{g}_t < s/c$ $\frac{10\%}{}$

$\Rightarrow I_t = 18$ ie $2(99-90)$.

$\Rightarrow Y_t = \frac{18}{0.2} = 90$. Here $\hat{g}_t = 9.09\%$

$s/c = 10\%$

Case B If $x_t = 101$. $\qquad \hat{g}_t > s/c$

$I_t = 2(101-90)$ $\quad 10.9\% > 10\%$

$= 22$

$Y_t = \frac{22}{0.2} = 110$.

Case A If they anticipate too little say 99 units of dd, Invst 'll be 18 units and actual demand Y_t will be 90.

Case B If they expect too much say 101 units of demand, then Invst 'll be 22 units to create capacity for 11 add'l units > the dd generated will be 110.

What will happen with $Y_t > X_t$?

$Y_t < X_t$

If actual dd Y_t < expected dd X_t, then underutilisation of capacity may results. But should $Y_t > X_t$, then how will the actual demand be met?

$Y_t > X_t$

One way is to dehoard the existing inventory but with Harrod's cumulative divergence from the warranted growth path for indefinitely, this can't be a case as the existing stock'll be exhausted.

Second way is to assume that any excess of Y_t over X_t is met by an unforseen rise in price level.

Transferring condition it cannot be explained.

Problem in the rise of price level.