



## Poverty and Inequality | Development Economics

### Important Formulae

$$\text{Mean} = \mu = \frac{1}{n} \sum_{j=1}^m n_j y_j$$

$$\text{Range} = R = \frac{1}{\mu} (y_{\max} - y_{\min})$$

$$\text{Kuznets ratio} = \frac{\text{Income of top } x\%}{\text{Income of bottom } x\%}$$

$$\text{Mean Absolute Variation} = M = \frac{1}{\mu n} \sum_{j=1}^m n_j |y_j - \mu|$$

$$\text{Coefficient of Variation} = C = \frac{1}{\mu} \sqrt{\sum_{j=1}^m \frac{n_j}{n} (y_j - \mu)^2}$$

$$\text{Gini Coefficient} = G = \frac{1}{2n^2\mu} \sum_{j=1}^m \sum_{k=1}^m n_j n_k |y_j - y_k|$$

$$\text{Headcount ratio} = \frac{\text{No of people poor}}{\text{Total no of people}}$$

$$\text{Poverty gap ratio} = \frac{\sum_{y_i < p} (p - y_i)}{nm} \text{ where } m \text{ is mean income}$$

$$\text{Income gap ratio} = \frac{\sum_{y_i < p} (p - y_i)}{p \cdot HC}$$

$$\text{Foster Greer Thorbecke index} = FGT = P_\alpha = \frac{1}{n} \sum \left( \frac{p - y_i}{p} \right)^\alpha$$

$$\text{If } \alpha = 0, FGT = HCR$$

$$\text{If } \alpha = 1, FGT = PGR$$

$$\text{If } \alpha = 2, FGT = HCR [IGR + (1 - IGR)^2 \cdot C] \text{ where } C \text{ is coefficient of variation}$$

Key:  $m$  distinct observation,  $y$  represents income class  $j$  for  $n_j$  number of individuals in each class,  $p$  refers to poverty line,  $i, j$  refers to individuals



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### Why should we care about inequality?

#### Intrinsic reason

Philosophical and ethical grounds:-  
there must be no reason why  
individuals should be treated  
unequally

#### Functional reason

Reduces the possibility of overall  
growth prospects of an economy

#### Anonymity

- doesn't matter who is earning the income
- permutations of income doesn't matter

#### Population principle

- cloning the entire population will not alter inequality provided the proportion remain the same
- i.e. size doesn't matter, proportion does

### 4 criteria for inequality measurement

#### Relative income price

- Relative income matters. absolute doesn't i.e. (2000, 4000) is same as (1000, 2000)
- Not same as population principle, there is strong assumption- utilities are proportional to income

#### Dalton principle

- Let  $(y_1, y_2, \dots, y_n)$  such that  $y_1 < y_2 < \dots < y_n$ .  
Let there be a regressive transfer ( i.e. a transfer from not richer to not poorer)
- The resultant new income distribution would be more unequal



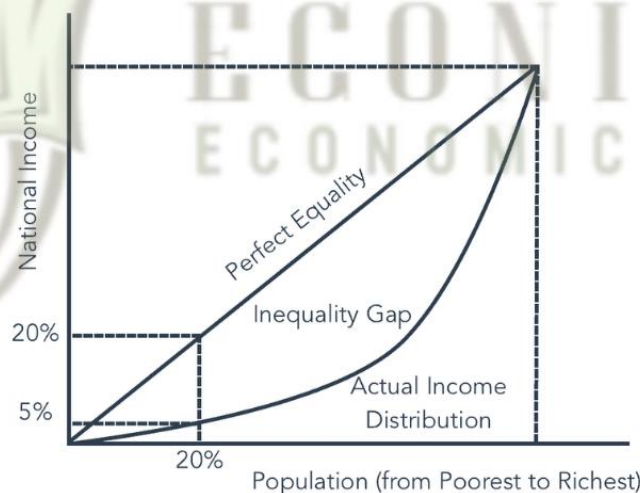
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## 4 principles of inequality index mathematically explained

Let the inequality index be represented by  $I$  with  $I = I(y_1, y_2, \dots, y_n)$

Principle	Mathematical condition
Anonymity	$I(y_1, y_2, \dots, y_n)$ for $y_1 < y_2 < \dots < y_n = I(y_n, y_{n-1}, \dots, y_1)$ for $y_n < y_{n-1} < \dots < y_1$
Population	$I(y_1, y_2, \dots, y_n) = I(y_1, y_2, \dots, y_n; y_1, y_2, \dots, y_n)$ for $y_1 < y_2 < \dots < y_n$
Relative	$I(y_1, y_2, \dots, y_n) = I(\lambda y_1, \lambda y_2, \dots, \lambda y_n)$ for $y_1 < y_2 < \dots < y_n$ and for $\lambda > 0$
Dalton	$I(y_1, y_2, \dots, y_i, \dots, y_j, \dots, y_n) < I(y_1, y_2, \dots, y_i - \delta, \dots, y_j + \delta, \dots, y_n)$ for $y_1 < \dots < y_i < \dots < y_j < \dots < y_n$

### Lorenz Curve



**Lorenz criterion:** An inequality measure  $I$  is Lorenz consistent if for every pairs of income distributions  $(y_1, y_2, \dots, y_n)$  and  $(z_1, z_2, \dots, z_m)$ ,  $I(y_1, y_2, \dots, y_n) \geq I(z_1, z_2, \dots, z_m)$  whenever lorenz curve of  $(y_1, y_2, \dots, y_n)$  lies everywhere to the right of  $(z_1, z_2, \dots, z_m)$ .

An inequality is Lorenz consistent iff it is simultaneously consistent with anonymity, population, relative income and Dalton principles.

A limitation of Lorenz curve is that we cannot compare two distributions if they cross each other.

Among all the inequality variables displayed on the first page, only coefficient of variation and Gini Coefficient are Lorenz consistent. Other (at least) fail the Dalton principle.





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## Lorenz Criterion

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### Overall expenditure or item by item consumption?

:- Dichotomy b/w Actual cons. basket vs overall exp i.e. cons. vs capacity to consume.

**Households or individuals?**:- Data might be available @ HH level with allocation within HH being skewed, problem of adult equivalence scaling and fixed cost of running a HH.

## Concerns regarding Poverty Measurement

**Absolute or Relative?**:- Some aspects are absolute like adequate level of food or clothing while other are relative like acceptable levels of participation in society, owning a car etc.

**Temporary or Chronic?**:- This are has significant fluctuations like economic shocks, differences in policies etc

**Why the need of poverty line anyway?**



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## About the Poverty measures

- HCR fails to capture the extent to which individual income falls below the poverty line. Those who are further away from the poverty line are poorer but HCR is insensitive to this info. Systematically policies are biased in favor of those near to the poverty line ('bang for the buck' problem).
- One way to offset the bias is to use a measure of average income shortfall from the poverty line which is done by PGR.
- But, PGR gives misleading impression of poverty in highly unequal but overall wealthy societies with a large number of poor people as m can be large enough to underestimate the PGR.
- To cater to this we have IGR which captures directly the acuteness of poverty because it measures relative to the total income needed to make that poverty go away. By ignoring the overall wealth of the society it tells us little about how easily the problem can be treated at least domestically. PGR/IGR avoids bang of the buck problem by neglecting numbers or fractions of people that are below the poverty line. It captures only the per capita intensity of poverty.
- Both HC and PG class of measures share an additional common drawback which is the issue of relative deprivation among poor. Both HCR and PGR also fails to satisfy the weak transfer principle.\*
- However, distributional sensitivity can be achieved by the variant of PGR (which is PGR') via raising the poverty gap by the power  $\alpha$  which gives us FGT, another robust measure of poverty. When there is no inequality, poverty can be expressed as a function of HCR and IGR from the FGT formula. Transfer sensitivity\*\* is satisfied by FGT iff  $\alpha > 2$ . FGT also has convenient decomposability property

\*Weak transfer principle:- A transfer of income from any person below the poverty line to anyone less poor while keeping the set of poor unchanged must raise poverty.

\*\*Principle of Transfer sensitivity:- A given regressive transfer between two people must matter more if incomes of persons involved are reduced equally.

*Note:- Formula of all measures mentioned are provided in the first page*





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Piketty and Saez- Income and Wealth Inequality from 'Inequality in the long run'

## Key historical trends

### Income Inequality:

- In the early 20th century, Europe exhibited higher income inequality than the United States, with the top 10% earning 45-50% of total income.
- After World War I and through the mid-20th century, European inequality declined significantly due to shocks like wars, depressions, and progressive policies. The top 10% income share fell to about 30%.
- Conversely, the U.S. saw a rise in income inequality after the 1970s, reaching nearly 50% by the 2010s.

### Wealth Inequality:

- Wealth inequality has consistently been more extreme than income inequality, with the top 10% historically controlling 60-90% of wealth.
- Pre-WWI Europe had virtually no "middle wealth class," whereas the U.S. maintained a wealth middle class controlling 20-30% of assets.
- Post-WWII Europe saw reduced wealth concentration, though it has since rebounded to about 65%.

### Wealth-to-Income Ratios:

- European economies show a U-shaped trend: high ratios pre-WWI, declines during the mid-20th century, and increases in the 21st century.
- U.S. wealth-to-income ratios are relatively stable, reflecting differences in growth rates and savings behavior.

## Mechanisms Behind Inequality

### Economic Growth and Capital Returns:

Wealth grows faster than economic output when the rate of return on capital ( $r$ ) exceeds economic growth ( $g$ ). Historically, this " $r > g$ " condition has driven wealth concentration. For example, slow growth rates in Europe (about 1-1.5%) combined with higher saving rates have led to high wealth-to-income ratios, whereas higher U.S. growth rates (2-3%) moderated capital accumulation.



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## Piketty and Saez- Income and Wealth Inequality from 'Inequality in the long run'

### Role of Shocks:

Major events, such as wars and economic crises, disrupted wealth accumulation in the early 20th century, particularly in Europe.

The recovery was slow due to physical destruction, public debt, and policy interventions (e.g., taxation and regulation).

### Elasticity of Substitution:

The relationship between capital and labor is influenced by the elasticity of substitution ( $\sigma$ ). A higher  $\sigma$  ( $>1$ ) suggests that capital can increasingly substitute labor, enhancing returns to wealth over wages and increasing inequality.

### Policy and Societal Implications

#### 1. Taxation:

- High taxes on capital and income significantly reduced inequality during the mid-20th century. Conversely, tax cuts since the 1980s have contributed to growing disparities.
- Progressive taxation, including wealth taxes, is proposed to counter rising inequality.

#### 2. Education and Technology:

- The "race between education and technology" shapes labor income inequality. Insufficient investment in education exacerbates skill gaps and wage disparities.
- The rise in top executive pay, especially in the U.S., further skews labor income distribution.

#### 3. Future Projections:

- With declining growth rates and increasing international tax competition, wealth concentration may intensify, potentially restoring "patrimonial capitalism."
- Policy responses, societal norms, and institutional frameworks will play critical roles in determining future inequality trajectories.