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It is also to be noted that Harrod's model is most suitable for developed economies.

Various modifications of the model has been done one of which was by P.C. Mahalanobis to make it more realistic and in line with developing economies. (refer lecture note)

Solow Model : (Neo classical model)

Assumptions

(a) Single homogeneous commodity is produced (say GDP can be treated as one)
One important implication is that there will be no international exchange.

(b). Technology is exogenous.
That is tech available to firms in this simple world is unaffected by the actions of the firms including R&D.

(c) We save a constant fraction s of the income $S = sY$ which goes to investment.

$$\Rightarrow C = Y - sY = (1-s)Y$$
$$= cY.$$

(d) We devote a constant fraction of time for acquiring skills and a constant fraction for production.

why price is kept const and @ ?

The basic solow model.

We have 2 main equations.
(i) Production fn. (ii) Capital Accumulation eqn.

Note that :- It is a neo classical model.
It is perf competition.
Price is constant.
Price of Y, L, K is constant.

Take $P=1 \Rightarrow$ real wage $= \frac{w}{P} = w$.
rental cost of capital $= r$.

Assume, the prod fn is Cobb Douglas

ie $Y = f(K, L)$ >> with only 2 inputs (simplification).
>> assumed to exhibit CRS.
>> linearly homogenous ie DoH = 1.

$$\boxed{Y = K^{\alpha} L^{1-\alpha}}$$

where α is output elasticity wrt capital.
$(1-\alpha)$ is " " " labor.

$0 < \alpha < 1$.

Optimization exercise.

$$\max_{K, L} F(K, L) - C.$$

$$\boxed{\max_{K,L} \ \pi = K^{\alpha}L^{1-\alpha} - \omega L - r K.}$$

$$\Rightarrow \frac{\partial Y}{\partial K} - r = 0 \quad \Rightarrow r = MP_K$$

$$\Rightarrow \frac{\partial Y}{\partial L} - \omega = 0 \quad \Rightarrow \omega = MP_L.$$

ie firms will hire labor and capital until the point where the MP_L and MP_K equals the cost of the input.

Further, the FOC means

$$r = MPK \text{ ie } \alpha K^{\alpha-1} L^{1-\alpha} = \alpha \, Y/K$$

$$\omega = MP_L \text{ ie } (1-\alpha) K^{\alpha} L^{-\alpha} = (1-\alpha)\frac{Y}{L},$$

From Euler's theorem

$$f'_x \cdot x + f'_y \cdot y = k \cdot f(x,y))$$

$$\Rightarrow \boxed{MP_K \cdot K + MP_L \cdot L} \quad — (1)$$

$$= \alpha \cdot \frac{Y}{\cancel{K}} \cdot \cancel{K} + (1-\alpha)\frac{Y}{\cancel{L}} \cdot \cancel{L}$$

$$= \alpha Y + (1-\alpha) Y$$

$$\boxed{= 1 \cdot Y.}$$

Thus α and $1-\alpha$ are also the share of output paid to capital & labor.

And for π maxn in this case $MP_K = r$, $MP_L = \omega$

Thus (1) becomes ~~with~~ $rK + \omega L$.

$= Y.$

ie payments to the inputs ie factor

payments completely exhaust the value of output produced so that there are no economic profits to be earned.

This important result is a general property of production fns with CRS.

This is the $\boxed{\text{product exhaustion theorm.}}$

Now, we are interested in output per worker and capital per worker.

Take $\boxed{y = \frac{Y}{L}}$ and $\boxed{k = \frac{K}{L}}$

$$Y = K^{\alpha} L^{1-\alpha}$$

$$\frac{Y}{L} = K^{\alpha} L^{-\alpha}$$

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^{\alpha}$$

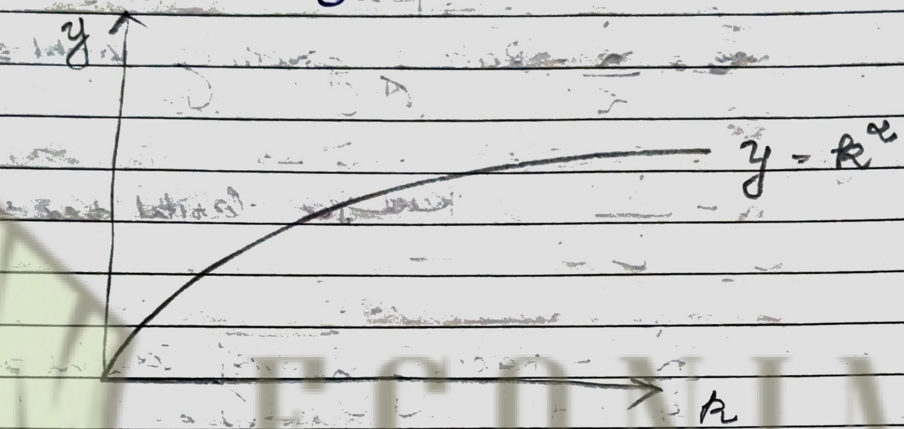
$$\Rightarrow \boxed{y = k^{\alpha}}$$

$k > 0$
$\alpha \in (0,1)$.

$$\frac{\partial y}{\partial k} = \alpha k^{\alpha-1} \qquad \frac{\alpha}{k^{1-\alpha}} > 0$$

ie as $k \to 0$
$\partial y/\partial k \to \infty$ and vice versa

This this production fn exhibit Diminishing returns to cpl per worker, each add'l unit of capital we give to a single worker ↑ the output but by a lesser and lesser rate.



Diminishing returns to cpl per worker.

$$\frac{dY}{dK} = \frac{dy}{dY} \cdot \frac{dY}{dK} \cdot \frac{dk}{dk}$$

$$= \frac{d(Y/L)}{dy} \cdot MP_K \cdot \frac{dk}{d(K/L)}$$

$$\Rightarrow \frac{dy}{dk} = \frac{1}{L} \cdot MP_K \cdot \frac{1}{1/L} \Rightarrow MP_K = \frac{dY}{dK}.$$

Hence DRS to capital per worker $\left\{ \begin{array}{l} \text{and we already know that as } \frac{dy}{dk} \to 0,\ k \to 0 \\ \text{and } \frac{\partial^2 y}{\partial k^2} < 0. \end{array} \right\}$

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Capital Accumulation equation

Remember :- K - capital stock.

$\dot{K} = \frac{dK}{dt}$ change in capital stock per period.

$\frac{\dot{K}}{K}$ - Rate of growth of capital stock.

$k = \frac{K}{L}$ - Rate ~~of~~ capital per worker

$\frac{\dot{k}}{k}$ - Rate of growth of capital per worker

Now, capital accumulation equation is given by

$$\boxed{\dot{K} = sY - dK}$$

$0 < s < 1$
$0 < d < 1$.

Thus there are 2 channels for change in capital stock :-

(a) ~~Savings~~ Gross investment. (sY)

Following Solow we assumed that constant fraction 's' will be saved and since Economy is closed it goes directly into investment and the only use of invest in this economy is to accumulate capital.

(b) Depreciation (dK)

We assume that a const fraction d of the cpl stock depreciates every period regardless of how much output is produced.

Now to rewrite this equation in capital per worker terms.

We know $k = \frac{K}{L}$

$\ln k = \ln K - \ln L$.

diff wrt t

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$$

$$\frac{sY}{K} - n - d$$

We already know $\frac{\dot{K}}{K} = \frac{sY - dK}{K}$

What about $\frac{\dot{L}}{L}$? Growth rate of labor force.

The growth rate of labor force $\frac{\dot{L}}{L}$ is assumed to be constant – by parameter n.

The exponential growth can be seen from the relationship

$$L(t) = L_0 e^{nt}.$$

So $\frac{\dot{k}}{k} \Rightarrow sY - dK$

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So $\dot{K} = sY - dK$.

$$\frac{\dot{K}}{K} = \frac{sY}{K} - d$$

from (#) $\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$

$$\frac{\dot{k}}{k} = \frac{sY}{K} - d - n.$$

$\Rightarrow$ ~~$\dot{k} = s$~~

$$\frac{\dot{k}}{k} = \frac{s\,Y/L}{K/L} - d - n.$$

(y in per labr term
similarly k)

$$\Rightarrow \frac{\dot{k}}{k} = \frac{sy}{k} - d - n$$

$$\Rightarrow \boxed{\dot{k} = sy - (n+d)k.}$$

So when it comes to capital per worker there are 3 channels of accumulation

Other than gross investment via savings and depreciation we have 'nk' term.

$\hat{k} = \frac{K\uparrow}{L}$

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Each period there are nL new workers around who were not there during last period. Cet par, capital per worker would decline because of the increase in labor force.

Often it is convenient describing the model to assume that labor force participation rate is unity i.e every member of popln is a worker.

Solving the basic Solow model.

We have

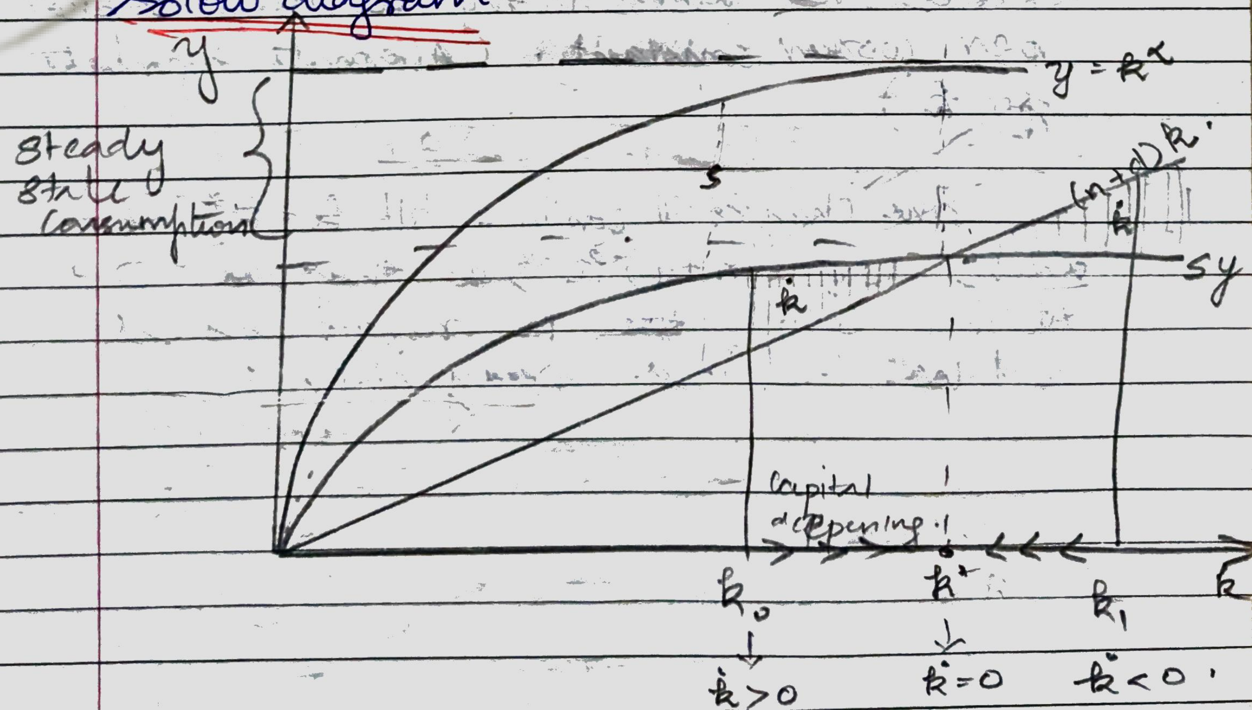
$$y = k^{\alpha}$$

and $\dot{k} = sy - (n+d)k$

$= sk^{\alpha} - (n+d)k.$

parameters: n, d, s, α
endogenous: y
exogenous: k, L

Solow diagram.



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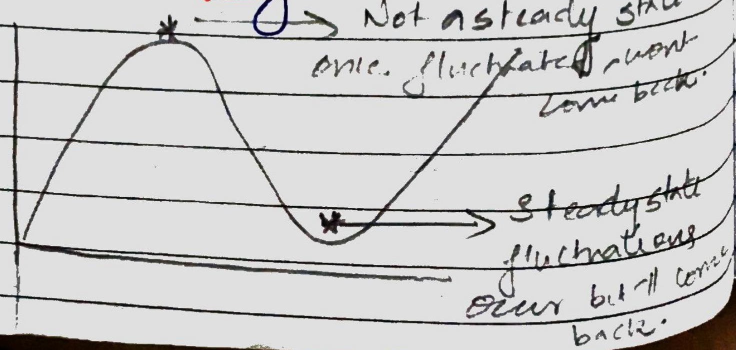
The solow diagram consists of mainly 2 curves (these 2 incorporated the prod fn as well). The first curve is the amt of invst per person, $sy = sk^{\alpha}$. This curve has the same shape as that of prod function with translated down by a factor of s.

The second curve is the line $(n+d)k$ which represents the amt of new invst per person required to keep the amt of capital per worker constant.

The diff b/w the two 2 curves is the change in the amt of capital per worker. When this change is positive and the economy is increasing its k, we say capital deepening is occuring. Movement from k_0 to k^* as at k_0 $\dot{k} > 0$ ie amt of invst per worker exceeds the amt needed to keep capital per worker constant. (Alternate arguments for k_1)

The changes 'll continue till $k = k^*$ at which $sy = (n+d)k$, so that $\dot{k} = 0$. At this pt, ~~the~~ k remains constant and we call it a steady state.



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Note:-
When this per worker change is zero ($\dot{k}=0$) but the actual capital stock K is growing (because of population growth) we say only capital widening is happening.

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$$

$$\dot{k}=0 \Rightarrow 0 = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} \Rightarrow \frac{\dot{K}}{K} = \frac{\dot{L}}{L}.$$

$$\text{Then } k = \frac{K}{L} = \frac{K'}{L'} = k$$

Also note from the diagram that steady state consumption is given by the diff b/w y curve and sy curve ~~at~~ corresponding to steady state -

$$\boxed{\dot{k} = sk^{\alpha} - (n+d)k}$$

At steady state ~~$\dot{k}=0$~~ $k = k^*$, $\dot{k}=0$

$$\Rightarrow sk^{*\alpha} = (n+d)k^*$$

$$k^{*\alpha-1} = \frac{n+d}{s}$$

$$k^{*1-\alpha} = \frac{s}{n+d}$$

$$\boxed{k^* = (s/n+d)^{\frac{1}{1-\alpha}}}$$

Thus
$$y^* = k^{*\alpha}$$

$$\boxed{y^* = \left(\frac{s}{n+d}\right)^{\frac{\alpha}{1-\alpha}}}$$

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We can solve for k using diff. eqn:

$$k = \left[\left(1 - e^{-\lambda t}\right)\frac{s}{n+d} + k_0^{1-\alpha} e^{-\lambda t}\right]^{\frac{1}{1-\alpha}}$$

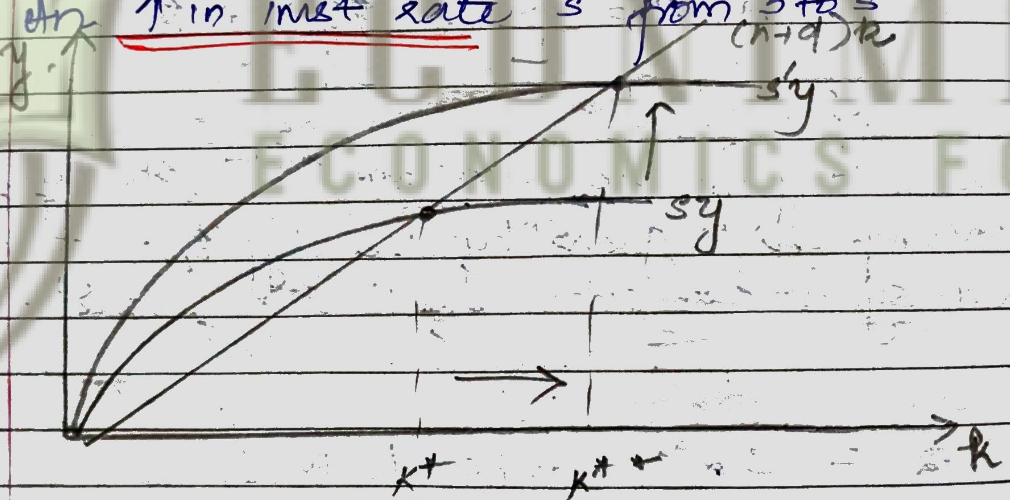
where $\lambda = (1-\alpha)(n+d)$

As $t \to 0$ $\quad k_t \to k_0$.

$t \to \infty$ $\quad k \to k^*$.

Comparitive Statics :

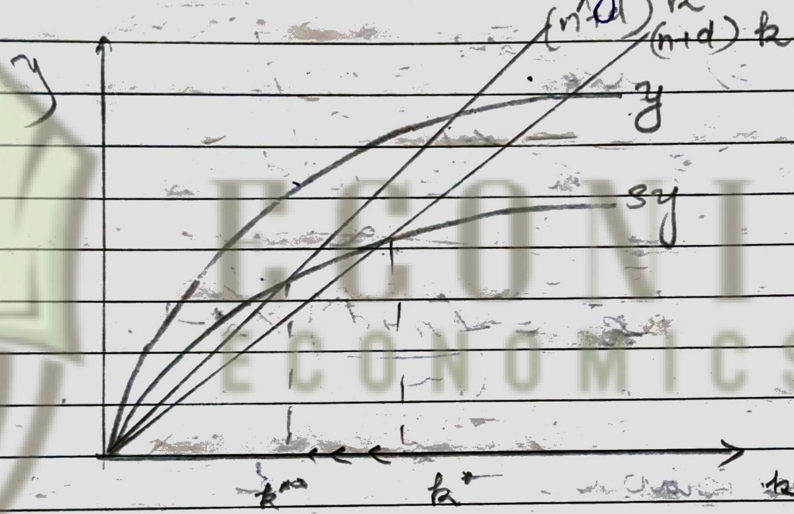
An ↑ in inst rate 's' from s to s'.



Increase in inst rate shifts the invst curve upward to s'y. At the current value of capital per worker ~~stock~~ (k^*), invest per worker now exceeds the amt required to keep capital per worker constant and therefore the economy begins capital deepening. Changes continue till k^{**}. Higher k^* means higher output. Economy is richer than before.

Increase in population growth:

If popln rises to n' from n : The $(n+d)k$ curve rotates up and to the left. At the current value of the capital stock k^*, investment per worker is now no longer high enough to keep the capital-labor ratio constant. Changes continue till k^{**}.



So the ultimate question of why are some countries richer and some poorer lies in the fact that. Higher s, lower n (and d) would make countries richer as higher would be their y and k.

Note :- If α is higher growth rate of output will be higher

But is a higher 's' to any extent desirable? We know $c+s=1$ and hence we should know that there exist a finite value b/w 0 and 1 for s where optimum consumption must also be met.

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Golden rule steady state :

$$c^* = k^{*\alpha} - (n+d)\, k^*$$

FOC: $$\frac{\partial c^*}{\partial k^*} = \alpha k^{*\alpha-1} - (n+d)$$

$$\frac{\partial c^*}{\partial k^*} = 0$$

$$\Rightarrow \alpha k^{*\alpha-1} = n+d$$

$$\Rightarrow k^{*\alpha-1} = \frac{n+d}{\alpha}$$

$$\boxed{k^* = \left(\frac{n+d}{\alpha}\right)^{1/\alpha-1}}$$

Now we know

$$k^* = \left(\frac{s}{n+d}\right)^{\frac{1}{1-\alpha}}$$

$$= \left(\frac{n+d}{s}\right)^{\frac{1}{\alpha-1}}$$

$$\Rightarrow \left(\frac{n+d}{s}\right)^{\frac{1}{\alpha-1}} = \left(\frac{n+d}{\alpha}\right)^{\frac{1}{\alpha-1}}$$

$$\Rightarrow \boxed{s = \alpha}$$

Economic Growth in the simple model.

We know $\dot{y} = \frac{dy}{dt}$ and rate of growth of $y = \frac{\dot{y}}{y}$.

$$y = k^{\alpha}$$

$$\ln y = \alpha \ln k$$

$$\boxed{\dot{y}/y = \alpha \frac{\dot{k}}{k}}$$

ie growth rate of output is some fraction of that of capital per worker.

Also $y = \frac{Y}{L}$

$$\ln y = \ln Y - \ln L$$

$$\Rightarrow \boxed{\alpha \cdot \frac{\dot{k}}{k} = \frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}}$$

We know $\dot{k} = sy - (n+d)k$.

$$\Rightarrow \frac{\dot{k}}{k} = \frac{s}{k^{1-\alpha}} - (n+d).$$

$$\Rightarrow \frac{\dot{y}}{y} = \alpha \cdot \frac{\dot{k}}{k}$$

$$\boxed{\frac{\dot{y}}{y} = \alpha \cdot \left[\frac{s}{k^{1-\alpha}} - (n+d)\right]}$$

Slope $= \frac{-s(1-\alpha)}{k^{1-\alpha}}$

when $k \to 0$
slope $\to \infty$.
and vice versa

$$\frac{\dot{k}}{k} = 0 \Rightarrow sk^{\alpha-1} = n+d.$$

Output per worker (y) [same as output per person i.e. since we assumed labor force participation rate is constant @ of growth of popln] is constant in the steady state.

Y is growing but at the rate of population growth

At the steady state $\frac{\dot{k}}{k} = 0$ and $\frac{\dot{y}}{y} = 0$, and they all grow @ n [the popl growth]

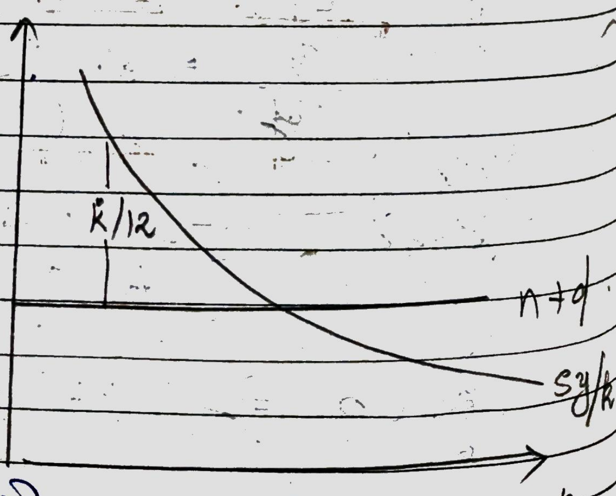
In this model the economies may grow for a while, but not forever.

If $k < k^*$, then the economy will experience growth in k and y along the transition path to the steady state. Over time however growth slows down as economy approaches steady state and eventually growth stops altogether

REMEMBER :- Level changes; growth doesn't.

Transition dynamics

The further an economy is from the steady state (below) faster its growth



(To see ~~why~~ the behavior of the lines read pg 36 first para)

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Technology and Solow Model.

To generate 'sustained growth', we must follow Solow and introduce technological progress. This is done by adding the tech variable A to our prod function.

$Y = F(K; L)$ with tech A can be represented in three ways

(i) $Y = A\,F(K, L)$ aka prod fn with Hicks-neutral technology.
(ii) $Y = F(AK, L)$ aka prod fn with capital augmenting or Solow neutral technology
(iii) $Y = F(K, AL)$ aka prod fn with labor augmenting or Harrod neutral technology

We choose (iii). why?

Taking $Y = F(AL, K) = K^{\alpha}(AL)^{1-\alpha}$.

$$\frac{Y}{L} = \frac{K^{\alpha}}{L^{\alpha}} \cdot A^{1-\alpha}.$$

$$y = k^{\alpha} A^{1-\alpha}.$$

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Assume the exogenous technological variable is going constant at the rate g.

i.e $\frac{\dot{A}}{A} = g \iff A(t) = A_0 e^{gt}$.

We know that $\frac{\dot{y}}{y} = \frac{d \ln y}{dt}$.

$$y = k^{\alpha} A^{1-\alpha}$$

So $\ln y = \alpha \ln k + (1-\alpha) \ln A$.

$$\frac{\dot{y}}{y} = \alpha \frac{\dot{k}}{k} + (1-\alpha) \frac{\dot{A}}{A} \quad \text{———} (*)$$

Capital accumulation equation

is $\frac{\dot{K}}{K} = \frac{sY}{K} - d$.

and $\frac{\dot{k}}{k} = \frac{sy}{k} - (n+d)$

Let $\frac{\dot{y}}{y} = \frac{\dot{k}}{k} = n$.

$\Rightarrow (*)$ becomes $n = \alpha \cdot n + (1-\alpha) \cdot g$.

$\Rightarrow n = g$

i.e $g_y = g_k = g_A$
$= g$.

#

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Remember that $\frac{\dot{K}}{K}$ is constant.

iff ~~Y and K are~~ constant i.e. $\frac{Y}{K}$ is constant.

If $\frac{Y}{K}$ is constant then $\frac{Y/L}{K/L}$ i.e. $\frac{y}{k}$ is constant

So $\frac{\dot{k}}{k}$ will also be constant.

A situation in which capital, output, consumption and population are growing at constant rates is called balanced growth ~~rate~~ path.

Now (ii) indicates that along a balanced growth path in the Solow model, y and k both grow at the rate of exogeneous technological change g.

The simple model ~~bare~~ with no tech progress and thus with no long run growth in y or k (i.e. y or k didn't experience sustained growth) was a special case of this model with $\frac{\dot{A}}{A} = 0$ hence $\frac{\dot{y}}{y} = 0$ and $\frac{\dot{k}}{k} = 0$.

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Solow Diagram with technology.

Here the variable k is no longer constant in the long run; ~~so we have to write~~

Thus new state variable 'll be $\tilde{k} = \frac{K}{AL}$

$\| ^{ly}$ $\tilde{y} = \frac{Y}{AL}$. i.e Ratio of capital (output) per worker to technology

K, K/L , $d\tilde{k}/dt$, K/AL | diff notation
k , $\frac{\dot{\tilde{k}}}{k}$, $\tilde{k}$

$\frac{\dot{\tilde{k}}}{\tilde{k}}$ $\Rightarrow \frac{d \ln \tilde{k}}{dt}$

$$\tilde{k} = \frac{K}{AL}$$

$$\Rightarrow \ln \tilde{k} = \ln K - \ln A - \ln L$$

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}.$$

$$\Rightarrow \frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{sY}{K} - d - g - n$$

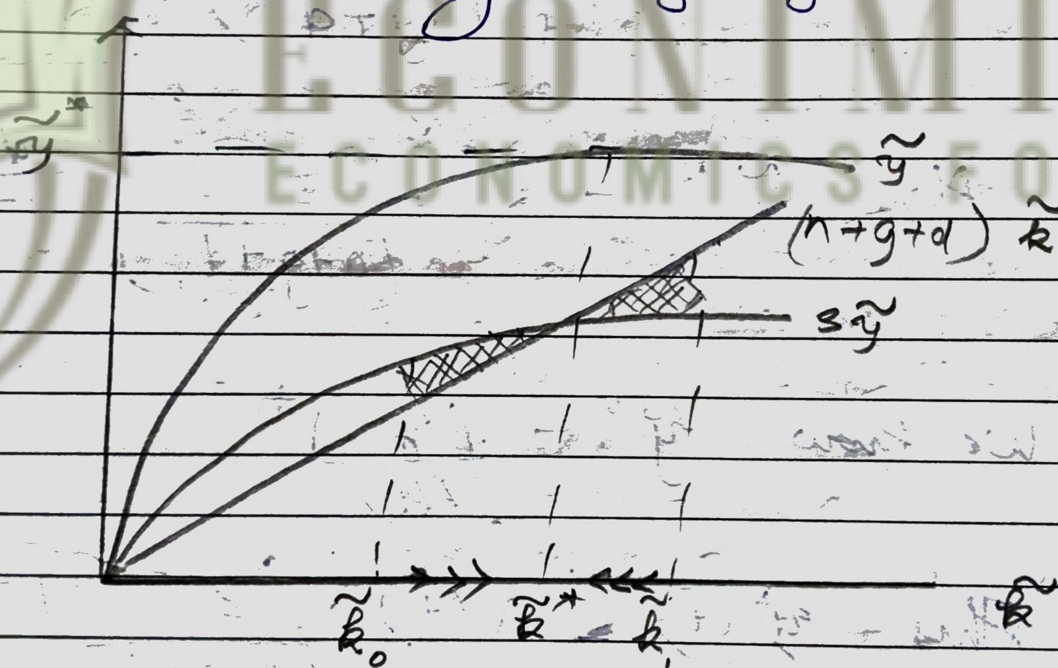
$$\Rightarrow \frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{sY}{K} - (n+g+d).$$

$$\Rightarrow \frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{s\,Y/AL}{K/AL} - (n+g+d)$$

$$= \frac{s\tilde{y}}{\tilde{k}} - (n+g+d)$$

$$\Rightarrow \dot{\tilde{k}} = s\tilde{y} - (n+g+d)\tilde{k}.$$

So here there are 4 channels. The new one included is that of technological growth 'g'.



why the curves are so (refer previous sections' expl - same).

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If the econy is @ $\tilde{k}_0$, then $\tilde{k}$ will rise over time as the amt of investment being undertaken (shown by $s\tilde{y}$ curve) exceeds the amt needed to keep $\tilde{k}$ constant. Change continue till $\tilde{k} = \tilde{k}^*$.

At steady state $\dot{\tilde{k}} = 0$.

$$\Rightarrow s\,\tilde{k}^{*\alpha} = (n+g+d)\,\tilde{k}^*$$

$$\Rightarrow \tilde{k}^* = \left(\frac{s}{n+g+d}\right)^{\frac{1}{1-\alpha}}.$$

Also $\tilde{y}^* = \tilde{k}^{*\alpha} = \left[\frac{s}{n+g+d}\right]^{\frac{\alpha}{1-\alpha}}.$

We know $\tilde{y}^* = \left(\frac{y}{A(t)}\right)^*$

Thus $y^*(t) = A(t) \cdot \left[\frac{s}{n+g+d}\right]^{\frac{\alpha}{1-\alpha}}.$

With special case of $g = 0$ and $A(t) = 1$ we will get the results of the simple Solow model.

$$y(t) = \left(\frac{s}{n+g+d}\right)(1-e^{-\lambda t}) + \left(\left(\frac{y_0}{A_0}\right)^{\frac{1-\alpha}{\alpha}} e^{-\lambda t}\right)^{\frac{\alpha}{1-\alpha}} \cdot A(t)$$

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So at balanced growth $\Rightarrow \frac{\dot{y}}{y} = \frac{\dot{k}}{k} = \frac{\dot{A}}{A} = g$.
(This was 0 in steady state of simple model)

We know $\tilde{y} = \tilde{k}^{\alpha}$

$$\Rightarrow \ln \tilde{y} = \alpha \ln \tilde{k}$$

$$\frac{\dot{\tilde{y}}}{\tilde{y}} = \alpha \left[\frac{\dot{\tilde{k}}}{\tilde{k}}\right] \quad \text{--- (*)}$$

ie growth rate of output per worker to tech is some fraction of that of capital per worker to tech.

We also know that in the model $y(t) = A(t) \cdot \tilde{y}$

ie $y = A \cdot \tilde{y}$

$$\Rightarrow \ln y = \ln \tilde{y} + \ln A.$$

$$\frac{\dot{y}}{y} = \frac{\dot{\tilde{y}}}{\tilde{y}} + \frac{\dot{A}}{A} \quad \text{--- (9)}$$

put (*) in (9)

$$\Rightarrow \alpha \cdot \frac{\dot{k}}{k} = \frac{\dot{\tilde{y}}}{\tilde{y}} + \frac{\dot{A}}{A}.$$

OR

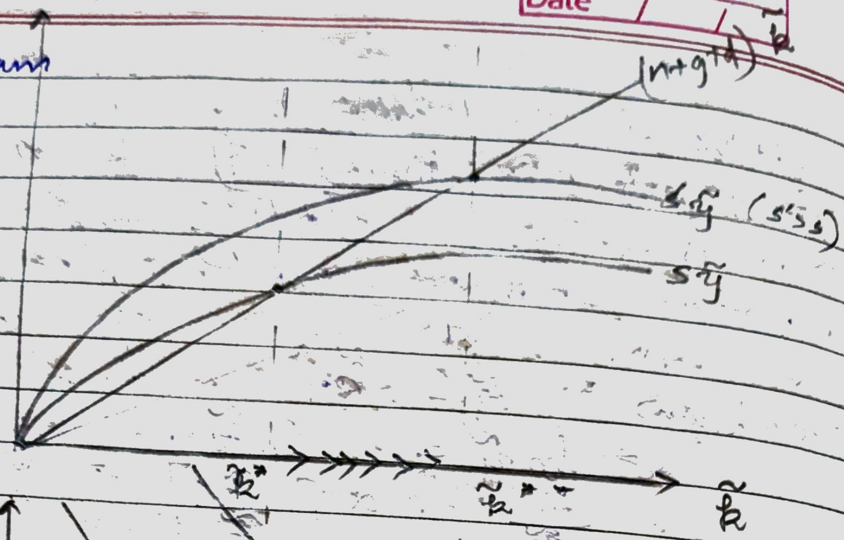
Take $y = k^{\alpha} A^{1-\alpha}$.

$$\Rightarrow \ln y = \alpha \ln k + (1-\alpha) \ln A.$$

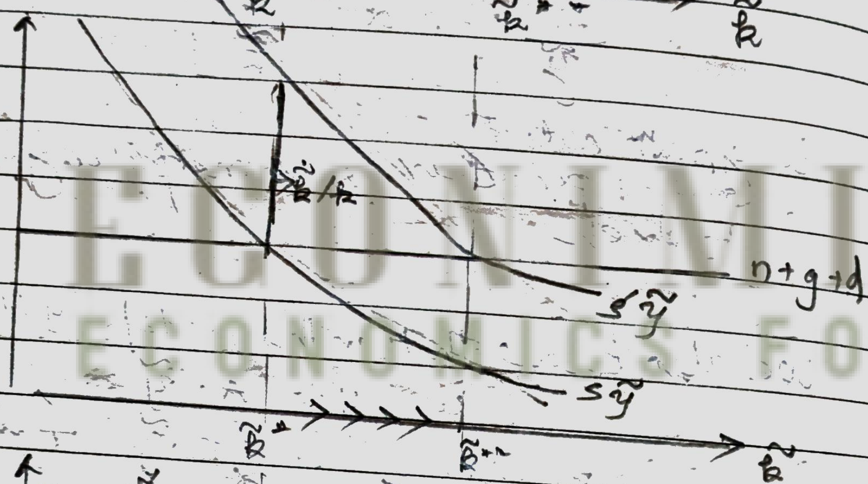
$$\Rightarrow \frac{\dot{y}}{y} = \alpha \frac{\dot{k}}{k} - (1-\alpha) \frac{\dot{A}}{A}.$$

Increase in savings or invst. rate.

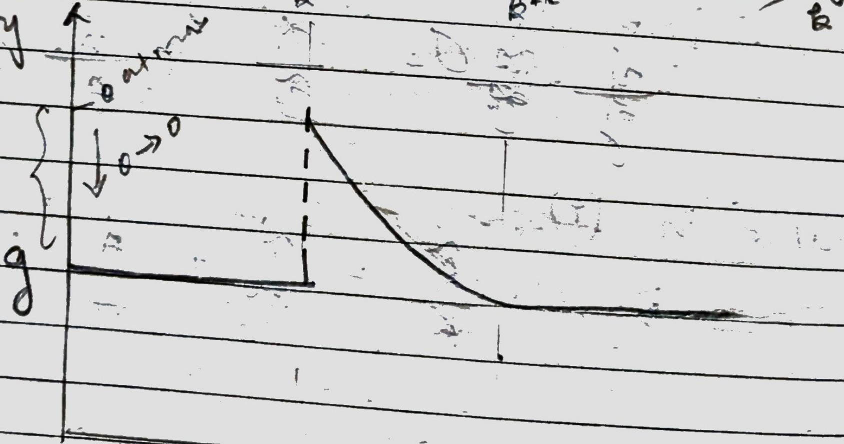
Solow Diagram



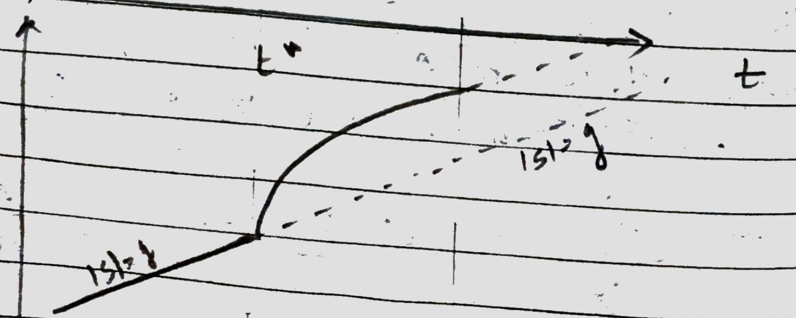
Transition Dynamics.



Behavior of Growth rate $\dot{y}/y$ of output.



Behavior of (log) level of output.



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Suppose an economy begins in steady state with investment rate s and then ↑ to s' forever.

So as seen from the ~~initial~~ Solow diagram, at the initial capital technology ratio $\tilde{k}^*$, investment exceeds the amt needed to keep the capital - tech. ratio const so $\tilde{k}$ rises till it reaches new steady state $\tilde{k}^{**}$.

The transition diagram shows that ↑ in s to s' raises the growth rate temporarily as the economy transits to the new steady state $\tilde{k}^{**}$. Since g is constant, faster growth in $\tilde{k}$ along the transition path implies that output per worker increases more rapidly than technology ie $\frac{\dot{y}}{y} = \theta + g$ rather than g. This behaviour y is shown in the third figure.

The last figure show how does this affect the 'level' of output per worker over time. Prior to the policy growth was at g, then the jump with $\theta + g$ and the value of $\theta \to 0$ and as we reach k^{**}, $\theta = 0$, $\dot{y}/y = g$ and again the growth rate is at g but we reached a higher level of output.

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Thus we can see that policy changes have no long run growth effect but only level effect. With tech (growth rate of g) or not (growth at 0 during steady state) a permanent policy change can permanently raise (or lower) the 'level' of per capita output.

Evaluating the Solow Model.

Solow model appeals to differences in invst rates and population growth rate and to exogenous diff in tech.

Acc to Solow model, ↓n, ↓d and ↑s etc can help us accumulate more k and thus ↑ labor productivity.

Why do economies exhibit sustained growth in Solow model? It is due to tech progress. Without tech progress, per capita growth will eventually cease as diminishing returns to capital set in. Tech progress can offset the tendency of MP_K to fall and in long run exhibit per capita growth at the rate of tech progress.

~~If it's costs~~

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Growth as we saw via transition ~~into~~ dynamics can happen either by a k further away from it pushed to the left of k^* or the steady state itself getting pushed right (say) to k^{**}. The first ~~one~~ reason is true for the growth spurt in countries like Japan and Germany after World War and second reason is suitable for NIC like South Korea and Taiwan.

Growth Accounting :

Prod. function :— $Y = BK^{\alpha}L^{1-\alpha}$ where B is a Hicks Neutral productivity term.

$$\Rightarrow \frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L} + \frac{\dot{B}}{B}$$

This growth a/c ing equation shows that output growth equals weighted avg of capital and labor plus growth rate of B.

$\frac{\dot{B}}{B}$ is known as Total Factor productivity or Multifactor productivity (TFP/MFP).

$$\text{Now } \frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}$$

$$= \alpha\frac{\dot{k}}{k} + \frac{\dot{B}}{B} \quad //.$$

Analysis of this eqn wrt US economy over the years — Read text.

Convergence and Principle of transition dynamics

Why do we see convergence among some sets of countries and lack of it for the world as a whole ?. (for context read 63-65).

Consider the key diff eqn of neo-classical growth model

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = s_k \frac{\tilde{y}}{\tilde{k}} - (n+g+d).$$

Remember $\tilde{y} = \tilde{k}^{\alpha} \Rightarrow$ Average product of capital is $\frac{\tilde{y}}{\tilde{k}} = \tilde{k}^{\alpha-1}$

$AP_k \downarrow$ as $\tilde{k} \uparrow$ because of diminishing returns to capital accumulation

So countries that have same steady state holds the convergence hypotheses that poor countries grow faster on average.

This was true for many models of growth hence we have the principle of transition dynamics :-

The further an economy is below its steady state, the faster the economy should grow. The further an economy is above its steady state, the slower the economy should grow.